Integration Background

# Indefinite Integrals:

In “pure” math, we are interested in the relationship between a function *f*(*x*) and its *integral* – the area beneath *f*(*x*). So we look for a function that describes the relationship using an *indefinite integral*, .

For simplicity, we will denote the results of such integration with functions that use the corresponding capital letter, so we say that we are looking for *F*(*x*) where *F*(*x*) = .

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| **Example 1:**  If the original function that we are looking at is:  *f*(*x*) =  we are looking for *F*(*x*) =  Using the rules of integration in calculus, we would find that:  *F*(*x*) =  =  =  *Technical note:* If we wish to be precise, the answer is:  *F*(*x*) =  for a constant *C*. | **Example 2:**  If the original function that we are looking at is:  *g*(*x*) =  we are looking for *G*(*x*) =  Using the rules of integration in calculus, we would find that:  *G*(*x*) =  =  =  *Technical note:* If we wish to be precise, the answer is:  *G*(*x*) =  for a constant *C*. |

# Definite Integrals:

In practical applications, we are generally looking for a specific number – the area underneath *f*(*x*) between two specific boundaries *a* and *b*. This is called a *definite integral* and is denoted by .

If we have already found the indefinite integral *F*(*x*) = , this area can be calculated by taking the difference between *F*(*b*) and *F*(*a*), so  = *F*(*b*) – *F*(*a*).

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| **Example 1:**  With *f*(*x*) as previously specified,  *f*(*x*) =  suppose we want to find the area under the curve between *x* = 6 and *x* = 8:  Then we are looking for:  =  =  Using *F*(*x*) =  as previously determined,  *F*(*x*) =  we have:  = *F*(8) – *F*(6) =  = 8 – 4.5  = 3.5 | **Example 2:**  With *g*(*x*) as previously specified,  *g*(*x*) =  suppose we want to find the area under the curve between *x* = 0 and *x* = 8:  Then we are looking for:  =  =  Using *G*(*x*) =  as previously determined,  *G*(*x*) =  we have:  = *G*(8) – *G*(0)  =  =  or approximately 29.87 |

The advantage of this approach is that we can easily calculate the area of integrals of the same function with different boundaries. Once we have figured out *F*(*x*) = , the only thing that we need to change are the values that we plug into *F*(*x*).

For instance, suppose that we now want to find the area under the curve between *x* = 0 and *x* = 8 in **Example 1** above. We could calculate this new integral as follows:

 = *F*(8) – *F*(0) =  = 8 – 0 = 8

# Computational Methods of Integration:

But what if *f*(*x*) is not easy to integrate – that is, *F*(*x*) is difficult to find or cannot be expressed in terms of elementary functions? (There are many such functions that are difficult to integrate.) Or what if we want to be able to integrate any arbitrary function *f*(*x*) without worrying about finding *F*(*x*)? What if we want a computerized solution for calculating the definite integral?

In such cases, we can determine  by using the computer to calculate the area beneath the curve of *f*(*x*) *instead* of finding *F*(*x*). There are a variety of techniques for doing so, but most of them follow the same general principle: break the interval [*a*, *b*] into small parts; find the approximate area under *f*(*x*) for each of the small parts; and add up the area of the parts to get an approximation for the desired integral.

What makes the techniques different from each other? There are different ways of finding the approximate area of the small parts, and these different ways give the techniques different properties. In addition, these techniques generally use iterative methods to get an approximation that is “close enough” to the actual value. We will focus on some of these computerized techniques, so this is as far as we will go with the underlying math.

# Example For Further Testing:

When we are examining the different techniques for integration, we should always begin by testing the techniques with integrals that have known results (so that we can be sure that the techniques are working correctly). Thus, here is one more example for use in further testing.

Suppose that we are interested in the curve defined by *f*(*x*) = . Then the appropriate indefinite integral is:

*F*(*x*) =  =  = 

If we are interested in the area under the curve between *x* = 3 and *x* = 9, we would calculate the definite integral as follows (see the diagram below on the left for a graphical representation):

 = 

= *F*(9) – *F*(3)

= () – ()

= (81 – ) – (3 – )

= 41.634 – 2.838

= 38.796

If we are interested in the area under the curve between *x* = 0 and *x* = 10, we would calculate the definite integral as follows (see the diagram below on the right for a graphical representation):

 = *F*(10) – *F*(0) =  – 0 =  ≈ 44.444444

 